Std. – 10th ICSE

Rathi's Aspire Academy
Preparing You For Tomorrow

Subject- Maths
Pre- Board Test Paper -1
Solution

SECTION A

1. (i) (d) ₹ 4720

Explanation: SP = ₹ 4000

$$\text{G.S.T.} = \frac{18}{100} \times 4000$$

$$= ₹\ 720$$

$\therefore$ Price paid = 4000 + 720

$$= ₹\ 4720$$

(ii) (b) 4

Explanation: $x^2 - kx + 4 = 0$

Have $a = 1, b = -k, c = 4$

For equal roots D = 0

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (-k)^2 - 4 \times 1 \times 4 = 0$

$\Rightarrow k^2 = 16$

$\Rightarrow k = 4$

(iii) (a) −5

Explanation:

Putting $x = \frac{1}{2}$ in $4x^2 + 8x + k$

$$\Rightarrow 4\left(\frac{1}{2}\right)^2 + 8\left(\frac{1}{2}\right) + k = 0$$

$$\Rightarrow 1 + 4 + k = 0$$

$$\Rightarrow k = -5$$

(iv) (a) 1 × 1

(v) (c) 9^{th}

Explanation: $a = 15$, $d = -2$, a_n = first negative term

$$\therefore \quad 0 > 15 + (n-1)(-2)$$

$$0 > 15 - 2n + 2$$

$$\Rightarrow \quad n > 8.5$$

$$\Rightarrow \quad n = 9$$

(vi) (b) (−7, 6)

(vii) (c) AAA

(viii) (a) 3 units

Explanation:

$$\frac{4}{3}\pi r^3 = 4\pi r^2$$

$$\Rightarrow \quad r = 3$$

(ix) (b) {3, 4, 5}

Explanation:

$$4 \le 2x - 2 < 10$$

$$6 \le 2x < 12$$

$$3 \le x < 6$$

$$\therefore \quad x = \{3, 4, 5\}$$

(x) (a) $\frac{1}{2}$

(xi) (a) 5

Explanation:

$$x + 2 \times 1 = 7$$

$$\Rightarrow \quad x = 5$$

(xii) (b) 30°

Explanation:

$$\angle A + 120° = 180°$$

$$\therefore \quad \angle A = 60°$$

In ΔABD : $90° + 60° \angle ABD = 180°$

$\therefore \angle ABD = 30°$

(xiii) (b) $y = 4$

Explanation:

Slope parallel to x-axis = 0

$\therefore$ P(3, 4) an $m = 0$

$$\Rightarrow \quad y - y_1 = m(x - x_1)$$

$$y - 4 = 0\ (x - 3)$$

$$\Rightarrow \quad y = 4$$

(xiv) (d) 3

Explanation:

$$a_n = 3x - 2$$

$$a_1 = 1$$

$$a_2 = 4$$

Common difference = 4 − 1 = 3

(xv) (b) 9 : 10

Explanation:

$$\frac{\text{Mode}}{\text{Median}} = \frac{6}{5} \qquad \frac{\text{Mean}}{\text{Median}} = ?$$

Mode = 3 Median − 2 Mean

Dividing throughout by median

$$\frac{\text{Mode}}{\text{Median}} = 3 - 2\frac{\text{Mean}}{\text{Median}}$$

$$\Rightarrow 2 \times \frac{\text{Mean}}{\text{Median}} = 3 - \frac{6}{5}$$

$$\Rightarrow \quad \frac{\text{Mean}}{\text{Median}} = \frac{9}{10}$$

2. (i) $P = ₹\,750$

$n = 24$ months

M.V. $= 19{,}125$

$r = ?$

$$\text{M.V.} = P \times n + \frac{Pn(n+1)r}{2400}$$

$$\Rightarrow \quad 19{,}125 = 750 \times 24 + \frac{750 \times 24 \times 25 \times r}{2400}$$

$$\Rightarrow 19{,}125 - 18000 = 187.5\, r$$

$$\Rightarrow \quad r = \frac{1125}{187.5}$$

$r = 6\%$ p.a.

(ii) $\dfrac{7m+2n}{7m-2n} = \dfrac{5}{3}$

Applying componendo and dividendo

$$\Rightarrow \frac{7m+2n+7m-2n}{7m+2n-7m+2n} = \frac{5+3}{5-3}$$

$$\Rightarrow \quad \frac{14m}{4n} = \frac{8}{2}$$

$$\Rightarrow \quad \frac{m}{n} = \frac{8}{7}$$

(iii) $\text{LHS} = \left(\dfrac{1}{\sin\theta} - \sin\theta\right)\left(\dfrac{1}{\cos\theta} - \cos\theta\right)\left(\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}\right)$

$$\Rightarrow \frac{(1-\sin^2\theta)}{\sin\theta} \times \frac{(1-\cos^2\theta)}{\cos\theta} \times \frac{(\sin^2\theta+\cos^2\theta)}{\sin\theta \times \cos\theta}$$

$$\Rightarrow \frac{\cos^2\theta}{\sin\theta} \times \frac{\sin^2\theta}{\cos\theta} \times \frac{1}{\sin\theta \times \cos\theta}$$

$= 1 = \text{RHS}$

3. (i) Volume of cylinder V_1

$$V_1 = \frac{22}{7} \times 3 \times 3 \times 7 \ \text{cm}^3 = 63\pi \ \text{cm}^3$$

Volume of cone V_2

$$V_2 = \frac{1}{3}\pi(3)^2 \times 3 \ = 9\pi \ \text{cm}^3$$

Volume of hemisphere

$$V_3 = \frac{2}{3}\pi(3)^3 \ = 18\pi \ \text{cm}^3$$

$\therefore$ Volume of remaining solid

$V_1 - V_2 - V_3 = (63\pi - 9\pi - 18\pi)\ \text{cm}^3$

$= 36\pi \ \text{cm}^3$

$$= 36 \times \frac{22}{7} = 113.14 \ \text{cm}^3$$

(ii) $3x + 4y = 12$

At x-axis $y = 0$

$\Rightarrow \quad 3x = 12$

or $\quad x = 4$

$P(4, 0)$

Slope of line $3x + 4y = 12$ is $-\dfrac{3}{4}$

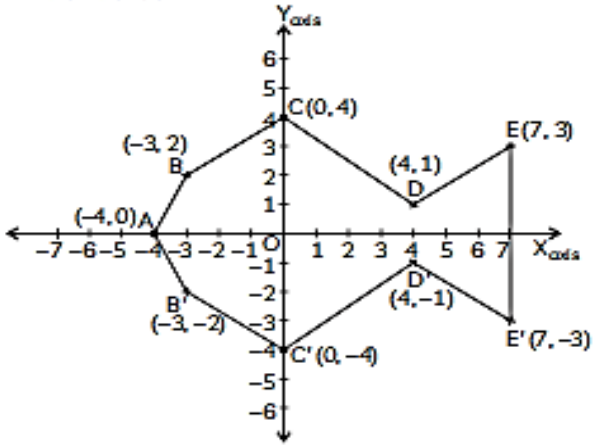
$\therefore$ Slope of $\perp$r line $= \dfrac{4}{3}$

Equation of line $y - y_1 = m(x - x_1)$

$$y - 0 = \frac{4}{3}(x-4)$$

$$3y = 4x - 16$$

(iii) (a), (c)



(b) $B'\ (-3, -2)$

$C'\ (0, -4)$

$D'\ (4, -1)$

$E'\ (7, -3)$

(d) Fish

SECTION B

4. (i) Chair : $1800 + \dfrac{18}{100} \times 1800 \ = ₹\ 2{,}124$

Vineer : $4000 + \dfrac{28}{100} \times 4000 \ = ₹\ 5{,}120$

Total bill $= ₹\ 7{,}244$

(ii) $x^2 + 3x - 4 = 0$

Here $a = 1, b = 3, c = -4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

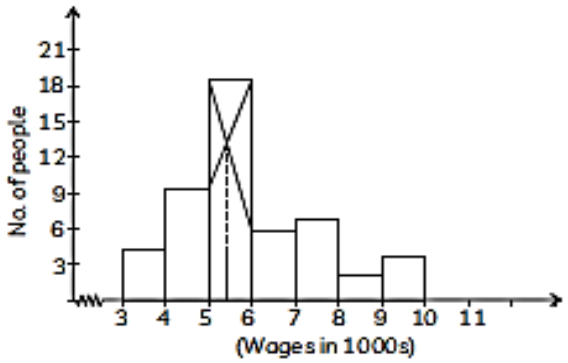
$$\Rightarrow x = \frac{-3 \pm \sqrt{(3)^2 - 4 \times 1(-4)}}{2 \times 1}$$

$\Rightarrow x = \frac{-3 \pm \sqrt{25}}{2}$

$\Rightarrow x = \frac{-3 \pm 5}{2}$

$\therefore x = -4, 1$

(iii)



Mode = 5,400

5. (i) $A^2 - 3I$

$\Rightarrow \begin{bmatrix} x & 3 \\ y & 3 \end{bmatrix}\begin{bmatrix} x & 3 \\ y & 3 \end{bmatrix} = 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x^2+3y & 3x+9 \\ xy+3y & 3y+9 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$\Rightarrow \quad 3x + 9 = 0$

$x = -3$

$3y + 9 = +3$

$\therefore \quad y = -2$

$\therefore x = -3$ and $y = -2$

(ii) (a) In ΔABC, $\angle B = 90°$

$\therefore 50° + 90° + \angle ACB = 180°$

$\Rightarrow \angle ACB = 40°$

(b) $\angle EAC = 40°$ (Alt. interior angles)

$\angle EAC + \angle D = 180°$

$\Rightarrow \angle D = 180° - 40°$

$\Rightarrow \angle EDC = 140°$

(c) $\angle BEC = \angle BAC$ (angle on the same arc)

$\Rightarrow \angle BEC = 50°$

(iii) $f(x) = 6x^3 + 17x^2 + 4x - 12$

Putting $x = -2$

$= 6(-2)^3 + 17(-2)^2 + 4(-2) - 12$

$= -48 + 68 - 8 - 12$

$= 0$

$\therefore x + 2$ is a factor of $f(x)$

$$x+2\overline{)\,6x^3+17x^2+4x-12\,(6x^2+5x-6}$$

$$\pm 6x^3 \pm 12x^2$$

$$5x^2 + 4x - 12$$

$$\pm 5x^2 \pm 10x$$

$$-6x - 12$$

$$\overset{-}{+}\ 6x \overset{+}{+}\ 12$$

$$\times$$

$\therefore f(x)(x + 2)(6x^2 + 5x - 6)$

$f(x) = (x + 2)[6x^2 + 9x - 4x - 6]$

$= (x + 2)[3x(2x + 3) - 2(2x + 3)]$

$= (x + 2)[(2x + 3)(3x - 2)]$

6. (i) (a) $A(-1, 3), B(4, 2), C(3, -2)$

Centroid $G\left(\frac{-1+4+3}{3}, \frac{3+2-2}{3}\right)$

$G(2, 1)$

(b) Slope of BC $= m = \frac{2-(-2)}{4-3} = 4$

Equation of line through G, parallel to BC

$y - y_1 = m(x - x_1)$

$\Rightarrow y - 1 = 4(x - 2)$

$\Rightarrow y - 1 = 4x - 8$

$\Rightarrow \quad y = 4x - 7$

(ii) LHS $= \sqrt{\sec^2\theta + \operatorname{cosec}^2\theta}$

$= \sqrt{\tan^2\theta + 1 + \cot^2\theta + 1}$

$= \sqrt{\tan^2\theta + \cot^2\theta + 2 \times \tan\theta.\cot\theta}$

$= \sqrt{(\tan\theta + \cot\theta)^2}$

$= \tan\theta + \cot\theta$

$=$ RHS

(iii) $a + 5d = 4a$

$\Rightarrow \quad 5d = 3a$ ...(i)

$\frac{6}{2}[2a + 5d] = 75$

$2a + 5d = 25$ ...(ii)

$5a = 25$ [from (i) & (ii)]

$a = 5$

$d = 3$

7. (i) (a) $\frac{13}{25}$

(b) {6, 12, 18, 24}

P(divisible by both 2 & 3) $= \frac{4}{25}$

(c) P(No. less than 16) $= \frac{15}{25} = \frac{3}{5}$

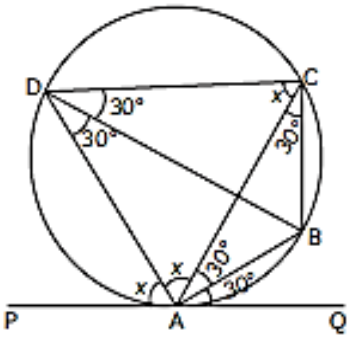
(ii) $V = \pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{22}{7} \times 7 \times 7 \times 4 + \frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$

$= 616 + 718.6$

$= 1334.6\ cm^3$

(iii) (a) Given: $\angle BAQ = 30°$



$\angle BDC = \angle BAC$

(Angle in the same segment)

$\angle BDA = 30°$

(Angle in alternate segment)

In ΔACD,

$60° + x + x = 180°$

$\Rightarrow \quad x = 60°$

Now $\angle BCD = 90°$ $(60° + 30°)$

$\therefore$ BD is a diameter.

(b) $\angle BCA = \angle BDA$ (angle in same segment)

$\angle BCA = 30° = \angle BAC$ $\therefore$ isosceles triangle

8. (i) $-2 + 10x \le 13x + 10 < 24 + 10x$

$-2 + 10x \le 13x + 10$

$-12 \le 3x$

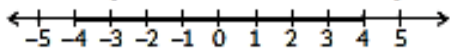
$-4 \le x$ ...(i)

$13x + 10 < 24 + 10x$

$3x < 14$

$x < \frac{14}{3}$ ...(ii)

$\therefore x \in \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$



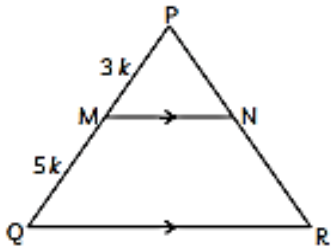
(ii) (a) $\frac{PM}{MQ} = \frac{3}{5}$

$PM = 3K$

$MQ = 5K$

$PQ = 3K + 5K = 8K$

In ΔPMN & ΔPQR



$\frac{PM}{PQ} = \frac{MN}{QR}$

$\frac{3K}{8K} = \frac{MN}{QR}$

$\therefore \quad \frac{MN}{QR} = \frac{3}{8}$

(b) $\angle PMN = \angle PQR$ (corresponding angles)

$\angle PNM = \angle PRQ$ (corresponding angles)

$\angle P = \angle P$ (comman)

$\therefore \Delta PMN \sim \Delta PQR$ (by AAA)

(iii)

x	f	fx
5	3	15
10	7	70
15	f	$15f$
20	9	180
25	6	150

$\Sigma f = 25 + f$

$\Sigma fx = 415 + 15f$

$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$

$16 = \frac{415 + 15f}{25 + f}$

$400 + 16f = 415 + 15f$

$f = 15$

9. (i) Let two natural numbers be x & y.

$x - y = 7$ ...(i)

$xy = 450$ ...(ii)

$(7 + y)\, y = 450$ [from (i) and (ii)]

$\Rightarrow \quad 7y + y^2 = 450$

$\Rightarrow y^2 + 7y - 450 = 0$

$\Rightarrow \quad y^2 + 25y - 18y - 450 = 0$

$\Rightarrow \quad y(y + 25) - 18(y + 25) = 0$

$\Rightarrow \quad (y + 25)(y - 18) = 0$

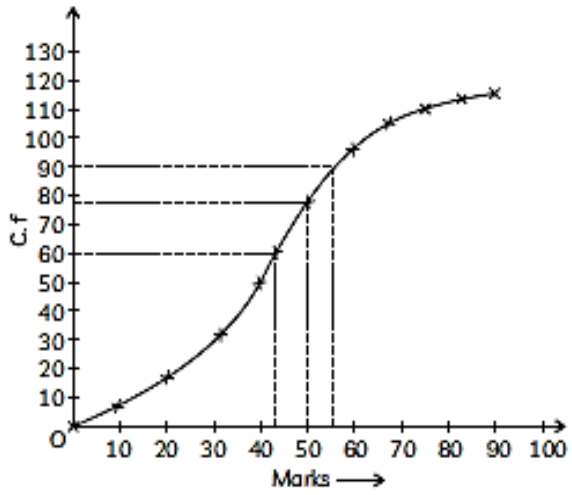
$\therefore \quad y = -25, 18$

for $y = 18$, $x = 25$

(ii)

Marks	f	c.f.
0 - 10	5	5
10 - 20	9	14
20 - 30	16	30
30 - 40	22	52
40 - 50	26	78
50 - 60	18	96
60 - 70	11	107
70 - 80	6	113
80 - 90	4	117
90 - 100	3	120

$\Sigma f = 120$



(a) Median $= \left(\frac{N}{2}\right)^{th}$ term $= \left(\frac{120}{2}\right)^{th}$ term $=$

60^{th} term $n = 120$

$\therefore$ Median $= 43$

(b) 78

(c) Upper quartile $(Q_3) = \left(\frac{3n}{4}\right)^{th}$ term

$= \left(\frac{3\times 120}{4}\right)^{th}$ term

$= 90^{th}$ term

$= 57$

10. (i) $\frac{\sqrt{5x}+\sqrt{2x-6}}{\sqrt{5x}-\sqrt{2x-6}} = \frac{4}{1}$

Applying componendo and dividendo

$$\frac{\sqrt{5x}+\sqrt{2x-6}+\sqrt{5x}-\sqrt{2x-6}}{\sqrt{5x}+\sqrt{2x-6}-\sqrt{5x}+\sqrt{2x-6}} = \frac{4+1}{4-1}$$

$$\frac{2\sqrt{5x}}{2\sqrt{2x-6}} = \frac{5}{3}$$

Squaring both sides

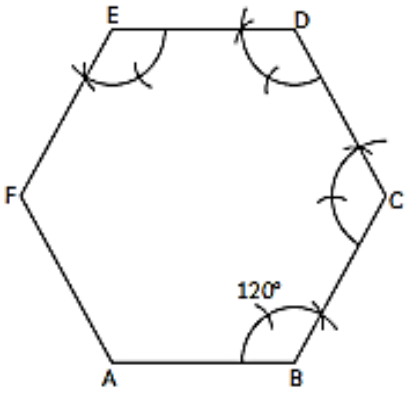
$$\frac{5x}{2x-6} = \frac{25}{9}$$

$$\Rightarrow \quad \frac{x}{2x-6} = \frac{5}{9}$$

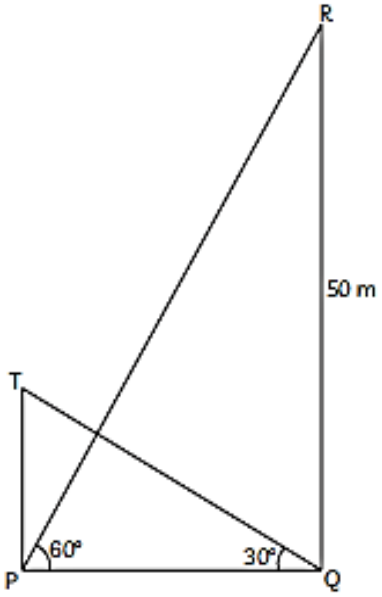
$$\Rightarrow \quad 9x = 10x - 30$$

$$\Rightarrow \quad x = 30$$

(ii)



(iii)



In ΔPQR,

$$\tan 60° = \frac{50}{PQ}$$

$$\Rightarrow \quad PQ = \frac{50}{\sqrt{3}}$$

In ΔPTQ,

$$\tan 30° = \frac{PT}{PQ}$$

$$\Rightarrow \quad PQ \times \frac{1}{\sqrt{3}} = PT$$

$$\Rightarrow \quad \frac{50}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = PT$$

$$\Rightarrow \quad PT = \frac{50}{3}$$

$\Rightarrow PT = 16.66$ m

$\Rightarrow PT = 17$ m